* Quadratic Equation *

①

Polynomial – An Expression of Type

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2}$ ——— a_0 is called Polynomial of degree n where all Power of x are nonnegative integer and a_n is called leading cofficient of the Polynomial not equal to 0.

Degree of the Eqn – It is the highest Power of the variable involved in the eqn.

$x^2 - 5x + 3$ degree – 2

$x^3 + 5x^2 - 7x + 6$ degree – 3

If leading cofficient is 1 then it is called monic Polynomial

Cubic Polynomial

$f(x) = ax^3 + bx^2 + cx + d$ $a \neq 0$

Quadratic Polynomial

$y = ax^2 + bx + c$ $a \neq 0$

a = leading Coffient

b = Cofficent of linear term

c = constant term/ absolute term.

Linear Polynomial $y = ax + b$ $a \neq 0$

Constant Polynomial $y = c$ $c \neq 0$

means $f(x) = 0$ It is not a Polynomial

If we equate any Polynomial $= 0$ then it will become equation.

$y = bx$ it is called odd linear Polynomial.

Quadratic Equation

$$ax^2+bx+c=0$$

Solution (roots) of the equation

① Factorisation method

$$ax^2+bx+c=0 \quad = a(x-\alpha)(x-\beta)$$

$$x^2-5x+6=0, \quad 3x^2-2x-1=0$$

② Shri Dharachrya Method

$$y = ax^2+bx+c=0$$

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

$$x = \frac{-b\pm\sqrt{D}}{2a} \qquad D = b^2-4ac$$

$$\alpha = \frac{-b+\sqrt{D}}{2a}, \qquad \beta = \frac{-b-\sqrt{D}}{2a}$$

Nature of Roots

$a, b, c \in R \quad a \neq 0$

① $D>0$ two distinct Real Root

(ii) $D=0$ real and equal

iii) $D<0$ root are Imaginary with non zero imaginary Part. $\sqrt{-1} = i \quad \sqrt{-4} = 2i$

iv) $D \geq 0$ for Real Root.

v) If one root is $\alpha+i\beta$ and other root will be $\alpha-i\beta$ $\quad 2+i3, \quad 2-3i$
always occur in Pair.

(vi) If cofficient of the QE are rational, $D \geq 0$ and Perfect Square, then both root are rational and distinct.

(vii) If $D > 0$ but not a Perfect square, If $P + \sqrt{Q}$
then other root will be $P - \sqrt{Q}$ (2)

<u>Sum of Roots</u> $= -b/a$

<u>Product of Root</u> $= c/a$

<u>Difference of Root</u> $= |\alpha - \beta| = \sqrt{(\alpha-\beta)^2}$

$$= \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$= \frac{\sqrt{D}}{|a|}$$

<u>Formation of QE when root are given</u>

Let α, β are roots

$(x-\alpha)(x-\beta) = 0$

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$x^2 - Sx + P = 0$

Q Solve the eqn $4^x - 5.2^x + 4 = 0$

Q Solve $3^{x^2-x} + 4^{x^2-x} = 25$

Q Solve $\sqrt{5x^2-6x+8} - \sqrt{5x^2-6x-7} = 1.$

Q find QE with rational cofficient whose one root is $2+\sqrt{3}$

Q Let α and β be the roots of Eqn $x^2+3x+1=0$
find $\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{1+\alpha}\right)^2$ (18)

Q If the sum of roots of the eqn $\frac{1}{x+a} + \frac{1}{x+b} + \frac{1}{c} = 0$
then P.T Product of the roots is $-\frac{1}{2}(a^2+b^2)$

$= x^2 + (a+b-2c)x + ab - bc - ca = 0$

$\alpha+\beta = -(a+b-2c) = 0$

$c = \frac{a+b}{2}$

$\alpha\beta = ab - bc - ca$

$= ab - c(a+b)$ $\quad ab - \left(\frac{a+b}{2}\right) a+b$

$= -\frac{1}{2}(a^2+b^2)$

Q Let α and β are root of $x^2+4x+1=0$

find (I) $\alpha^2+\beta^2$ (II) $\alpha^2-\beta^2$ (III) $\alpha^3+\beta^3$ (4) $\alpha^4+\beta^4$

$= (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2$

Q If the ratio of root of the eq^n $x^2+px+q=0$ are equal to the ratio of the roots of $x^2+bx+c=0$ then P.T $p^2c = b^2q$

Q Find the value of a for which one root of the QE $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as other

Q If α, β are the roots of Eq^n $x^2 - p(x+1) - q = 0$ then value of

$$\frac{\alpha^2+2\alpha+1}{\alpha^2+2\alpha+q} + \frac{\beta^2+2\beta+1}{\beta^2+2\beta+q}$$

Q If roots of the QE $ax^2+bx+c=0$ is α and β

find the QE whose roots are

(I) $\frac{1}{\alpha}, \frac{1}{\beta}$ (II) α^2, β^2 (III) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ (IV) $\alpha+2, \beta+2$

(I) $S = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-b}{a}\cdot\frac{a}{c} = -\frac{b}{c}$

$P = \frac{1}{\alpha\beta} = \frac{a}{c}$ $\quad x^2 - (-\frac{b}{c})x + \frac{a}{c} = 0$ $\boxed{cx^2+bx+a=0}$

(II) method

$\frac{1}{\alpha} = x \qquad \alpha = \frac{1}{x}$

$a\alpha^2 + b\alpha + c = 0$

$a\frac{1}{x^2} + \frac{b}{x} + c = 0 \qquad \boxed{cx^2 + bx + a = 0}$

(ii) α^2, β^2

$a\alpha^2 + b\alpha + c = 0$

$\alpha^2 = x \qquad \alpha = \sqrt{x}$

$ax + b\sqrt{x} + c = 0$

$ax + c = -b\sqrt{x}$

$a^2x^2 + c^2 + 2acx = b^2x$

$a^2x + (2ac - b^2)x + c^2 = 0$

$S = \alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2 - 2ac}{a^2}$

$P = \alpha^2\beta^2$

$= \frac{c^2}{a^2}$

$x^2 - \left(\frac{b^2 - 2ac}{a^2}\right)x + \frac{c^2}{a^2}$

$= \underline{ax^2 + (2ac - b^2)x + c^2 = 0}$

(iii) $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \qquad \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{b^2 - 2ac}{a^2 . c} . a$

$x^2 - Sx + P = 0$

$P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$x^2 - \left(\frac{b^2 - 2ac}{ac}\right)x + 1$

(iv) $S = \alpha + 2 + \beta + 2$

$= \alpha + \beta + 4$

$= -b/a + 4$

$P = (\alpha+2)(\beta+2)$

$= \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha+\beta) + 4$

$= c/a + 2\frac{-b}{a} + 4$

(II) method

$a\alpha^2 + b\alpha + c = 0$

$x = \alpha + 2$

$\alpha = x - 2$

$a(x-2)^2 + b(x-2) + c = 0$

Q. If a, b, c are in AP and one root of eqn $ax^2+bx+c=0$ is 2 find other.

Q. Given both root of eqn $mx^2+(2m-1)x+m-2=0$ are rational. Find the set of all +ve integral values of m.

= $D = 4m+1$

$m = 2, 6, 12, 30$

= $m = n(n+1)$ $\boxed{n \in N}$

Q. $p(q-r)x^2 + q(r-p)x + r(p-q) = 0$ has equal root, then p, q, r in

= one root is 1 other root also will be 1 so

$$\frac{r(p-q)}{p(q-r)} = 1$$

Q. If one root of eqn $3x^2+px+3=0$ where $p>0$ and one root is square of other, find p.

Q. Solve $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$

Q. If α, β are root of the roots of the Eqn $x^2-6x-2=0$ where $\alpha>\beta$ and $a_n = \alpha^n - \beta^n$ then find $\frac{a_{10} - 2a_8}{2a_9}$ is

Q. If α, β are the roots of Eqn $x^2-2x+5=0$, then form a QE whose roots are $\alpha^3+\alpha^2-\alpha+22$ and $\beta^3+4\beta^2-7\beta+35$

Q. Find roots of the Eqn $(x^2+x-2)(x^2+x-3)=12$

Q. Find the integral value of a such that roots of the eqn $(x-a)(x-10)+1=0$ are integer roots.

Q. If $\sin\alpha$ and $\cos\alpha$ are roots of the equation $ax^2+bx+c=0$ then,

a) $a^2+b^2=2ac$ B) $a^2-b^2=2ac$

c) $a^2-b^2=-2ac$ d) $a^2+b^2=c^2$

$$= \sin\alpha+\cos\alpha = -b/a$$

$$1+2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$$

$$1+2\frac{c}{a} = \frac{b^2}{a^2}$$

Q. If the roots of $ax^2+bx+c=0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$ then find the roots of $a^3x^2+abcx+c^2=0$ in term of α, β.

Note* If $ax^2+bx+c=0$

and $a+b+c=0$ then $x=1$ will be the root of the equation.

Q. Solve $4^x+6^x=9^x$

$$= \left(\frac{4}{9}\right)^x+\left(\frac{6}{9}\right)^x = 1$$

$$= \left(\frac{2}{3}\right)^{2x}+\left(\frac{2}{3}\right)^x = 1 \qquad t^2+t-1=0$$

Q. If $x=1+2i$ find x^3+x^2-x+22

* Graphs of QE

$$y = ax^2+bx+c$$

$$y+\frac{D}{4a} = a\left(x+\frac{b}{2a}\right)^2$$

$$Y = ax^2$$

If $a>0$ ∪ $a<0$ ∩

vertex of Parabola $\left(-b/2a, \frac{-D}{4a}\right)$

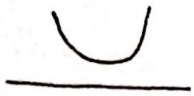
$a>0 \quad d>0$

$y > 0$

$(-\infty\ \alpha) \cup (\beta\ \infty)$

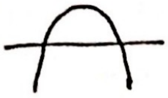
$a>0 \quad d=0$

$y \geq 0$



$a>0 \quad d<0$

$y>0$



$a<0 \quad d>0$

$y>0$

$y \in (\alpha\ \beta)$

$a<0 \quad d=0$

$y \leq 0$

$a<0 \quad d<0$

$y<0$

(double derivative -ve aayega to concave down)

Q Draw graph of $y = x^2 - 7x + 12$, then find the value of x where y is +ve

Q The Q.E. $ax^2 + bx + c = 0$ has no real root then prove that $c(a+b+c) > 0$

Q If $a, b, c, d \in R$ such that $a<b<c<d$ then show that root of $(x-a)(x-c) + 2(x-b)(x-d) = 0$ are real and distinct

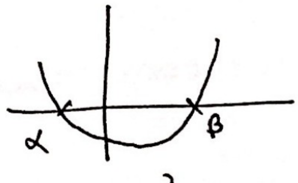
Q If the roots of the eqn $x^2 - 8x + a^2 - 6a = 0$ are real then find the interval of a.

Q If the difference between the roots of eqn $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then set of possible value of a is

Q find the sign of

① s ② p c) a d) b e) c f) d

$a>0$



$y = ax^2 + bx + c$

$\boxed{P<0}$ $\boxed{S>0}$ $\boxed{a>0}$

$f(0) = c$ $\boxed{c<0}$

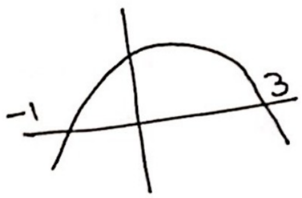
$b^2 - 4ac > 0$ $\boxed{D>0}$

$S = -\frac{b}{a} > 0$

$= -b > 0$

$\boxed{b<0}$

Q Consider the Graph $y = ax^2 + bx + c$



a) $\frac{a-b+c}{abc} = 0$

b) $abc(9a+3b+c) < 0$

c) $\frac{a+3b+9c}{abc} < 0$

d) $ab(a-3b+9c) > 0$

$=$ $a<0$ c $\alpha\beta = P < 0$

$c/a < 0$ $c>0$

vertex $-\frac{b}{2a} > 0$ $\boxed{b>0}$

so $abc < 0$

$f(-1) = a - b + c$

$f(\frac{1}{3}) > 0$ $f(3) = 9a + 3b + c = 0$

$a + 3b + 9c > 0$ $f(-\frac{1}{3}) > 0$ $a - 3b + 9c > 0$

Q If one root is square of other root of the eqn $x^2 + px + q = 0$, then find the relation between p and q is

Q If α, β be the root of the eqn $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ be the root of the eqn $x^2 - qx + r = 0$ then value of r is

Q the no of Point of intersection of the curve $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is

Q Let $f(x)$ be a Quadratic Expression which is +ve for all real x

If $g(x) = f(x) + f'(x) + f''(x)$ then for all real x

a) $g(x) < 0$ B) $g(x) > 0$ c) $g(x) = 0$ d) $g(x) \geq 0$

Q Find the set of values of a for which the quadratic Polynomial

$(a+4)x^2 - 2ax + 2a - 6 < 0 \quad \forall \; x \in R$

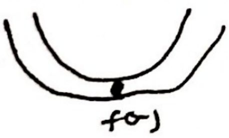
Q Let $P(x)$ be a Quadratic Polynomial with real +ve cofficient satisfying

$x^2 - 2x + 2 \leq f(x) \leq 2x^2 - 4x + 3 \quad \forall \; x \in R$ and

$f(11) = 181$ find $f(16)$

$g(x)$ veltex $(1\,1)$

$h(x)$ veltex (11)



$f(x) = \alpha(x-1)^2 - 1$

$x = 11$

$\alpha = 9/5$

$f(x) = \frac{9}{5}(x-1)^2 - 1$

$f(16) = \underline{406}$

Solving Q.E and Rational Inequality

wavy curve method

1) factorise given Expression into linear factor
2) Make cofficient of x Positive in all factor

③ Plot the zeroes on no line in increasing order.
④ Start no line from right to left taking +ve -ve alternate.

(T-1) Non Repeated linear factor

Q $(x-1)(x-2)(x-3) \geq 0$

Q $(x^2-x-6)(x^2+6x) \geq 0$

(T-2) Repeated linear factor

Q $(x+1)(x-3)(x-2)^2 \geq 0$

Q $x(x+6)(x+2)^2(x-3) > 0$

Q $(x-1)^2(x+1)^3(x-4) < 0$

(T-3) inequality Involving $\frac{f(x)}{g(x)}$

Q $\frac{2x-3}{3x-7} > 0$

Q $\frac{x^3(2x-3)^2(x-4)^6}{(x-3)^3(3x-8)^4} \leq 0$

Q $\frac{x^2-5x+12}{x^2-4x+5} > 3$

D<0 cross Multiply

Q $\frac{x^2-5x+6}{x^2+x+1} < 0$

D<0

Q $\frac{(x-1)^2(x+1)^3}{x^4(x-2)} < 0$

Q $\frac{x+1}{x-1} \geq \frac{x+5}{x+1}$

Q $\frac{2(x-4)}{(x-1)(x-7)} \geq \frac{1}{x-2}$

Q $\frac{x^2+6x-7}{|x+4|} < 0$

always +ve cross Multiply

Q $1 < \frac{3x^2-7x+8}{x^2+1} \leq 2$

Q Solve $(x^2+3x+1)(x^2+3x-3) \ge 5$

$\cancel{(t+1)(t-3) \ge 5}$

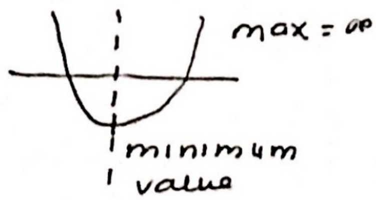
$t(t-4) \ge 5 \qquad (t-5)(t-1) \ge 0$

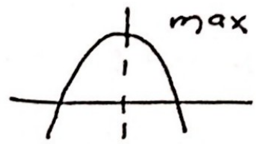
$(x+4)(x-1)(x+1)(x+2) \ge 0$

* Range of QE

$f(x) = ax^2+bx+c$

$a>0$ $\qquad\qquad$ $a<0$





$= a\left[\left(x+\frac{b}{2a}\right)^2 - \frac{b^2-4ac}{4a^2}\right]$

$\left(-\frac{b}{2a}, \frac{-D}{4a}\right)$ vertex $\quad$ yaha par the maxima ya minimum aayega ye a par depend karega

Q Find max value of $-3x^2+6x+5$

Q Let $P(x) = ax^2+bx+8$ is a Quadratic Polynomial if the minimum value of $P(x)$ is 6 when $x=2$ find a and b.

Q For $x \ge 0$ what is smallest Possible value of the expression

$\log_{10}(x^3-4x^2+x+26) - \log_{10}(x+2)$.

$= x^3-4x^2+x+26 > 0 \qquad x+2>0$

$(x+2)\underbrace{(x^2-6x+13)}_{+ve} > 0$

$x+2>0$

$x>-2$

$y = \log_{10}\left(\frac{\cancel{(x+2)}(x^2-6x+13)}{\cancel{(x+2)}}\right)$

$= \log_{10}((x-3)^2+4)$

$= \log_{10}4$

Q. $f(x) = x^2 - 4x + 7$ find Range if

(i) $x \in R$

(ii) $x \in [-3, 1]$

iii) $x \in [0\ 5]$

iv) $x \in [3, 5]$

(i) $x^2 - 4x + 4 + 7 - 4$ $\qquad (x-2)^2 + 3$

$[3\ \infty)$ $\qquad$ = minimum at $x = 2$ will be 3.

ii) 2 is not lying so one will give minima and othe maxima

$f(-3) = 9 + 12 + 7 = 28$ $\qquad [4\ \ 28]$

$f(1) = 1 - 4 + 7 = 4$

iii) 2 is lying

$f(0) = 7$

$f(5) = 25 - 20 + 7 = 12$ $\qquad [3\ \ 12]$

$f(2) = 4 - 8 + 7 = 3$

(iv) $(3\ \ 5]$

$f(3) = 9 - 12 + 7 = 4$ $\qquad [4\ \ 12]$

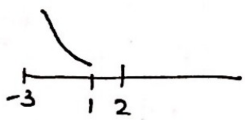
$f(5) = 12$

(II) method (I) $f'(x) = 2x - 4 = 0$ $\qquad \underline{x = 2}$

$f'(x)$ sign scheme $\qquad - \cup +$ at 2 $\qquad$ minima

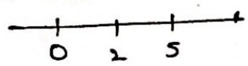
minimum of $f(x) = 4 - 8 + 7 = 3$ at $x = 2$

(ii) $[f(1)\ \ \ f(-3)]$



(iii) minimum at 2

max 0 or 5



$f(2) = 3$

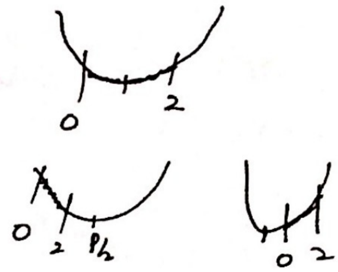
$f(0) = 7$

$f(5) = 25 - 20 + 7 = 12$ $\qquad [3\ 12]$

(iv) $[f(3) \quad f(5)]$

Q. find the value of P for each of which least value of Polynomial $f(x) = 4x^2 - 4Px + P^2 - 2P + 2$ on the interval $x \in [0\ 2]$ is equal to 3.

$$x = -\frac{b}{2a} = \frac{4P}{4.2} = \frac{P}{2}$$



case 1 vertex

$$0 \le \frac{P}{2} \le 2$$

$$\boxed{0 \le P \le 4}$$

$$\frac{-D}{4a} = 3$$

$$\frac{-(16P^2 - 4.4.(P^2 - 2P + 2))}{16} = 3$$

$P = -\frac{1}{2}$ × Not Possible

Case-2 $\quad \frac{P}{2} > 2 \qquad P > 4$

$$f(2) = 3$$

$$16 - 8P + P^2 - 2P + 2 = 3$$

$P = 5 \pm \sqrt{10}$ $\quad \boxed{P = 5 + \sqrt{10}}$

case-3 $\quad \frac{P}{2} < 0 \qquad P < 0$

$$f(0) = 3$$

$$P^2 - 2P + 2 = 3$$

$$P^2 - 2P - 1 = 0$$

$$(P-1)^2 = 2$$

$$P = 1 \pm \sqrt{2}$$

$$\boxed{P = 1 - \sqrt{2}}$$

Range of Rational function

$$y = \frac{Q}{L} = \frac{Q}{Q} = \frac{L}{Q} = \frac{L}{L}$$

Note $y = \dfrac{(x-a)(x-b)}{x-c}$

if $a < c < b$

then Range of $\boxed{y \in R}$

Note $y = \dfrac{(x-a)(x-b)}{(x-c)(x-d)}$

exact one of c and d

lie between a and b then range is R.

Q $\dfrac{x+3}{x-2}$ Q $y = \dfrac{3x+2}{x-1}$ Q $y = \dfrac{(x-1)(x-2)(x-3)}{(x-2)(x-3)}$

Q Range of $y = \dfrac{x^2 - 3x + 4}{x^2 + 3x + 4}$

Also check $y = 1$ hoga ya Nahi.

$$\cancel{x^2} + 3x + 4 = \cancel{x^2} - 3x + 4$$

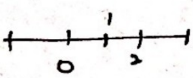
$$6x = 0 \quad \boxed{x = 0}$$

$$y \in \left[\frac{1}{7} \quad 7\right]$$

Q $y = \dfrac{(x+1)(x-2)}{x(x+3)} = R$ $\boxed{\text{direct}}$

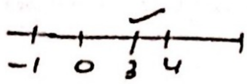
Q $y = \dfrac{x^2 - x + 1}{x^2 + x + 1}$ $y \in \left[\frac{1}{3} \quad 3\right]$

Q $y = \dfrac{x-1}{x(x-2)}$



R direct

Q $y = \dfrac{x(x-4)}{(x+1)(x-3)}$



$\boxed{y \in R}$

Q $y = \dfrac{x^2+x+2}{x^2+x+1}$

$y=1$ Nahi hoga
check karne

Q Find all possible value of a for which expression $\dfrac{ax^2-7x+5}{5x^2-7x+a}$ may be capable of all values of x being real. and range is $(-\infty\ \infty)$

$D \geq 0$

$y^2(49-20a) + 2y(1+2a^2) + 49-2a \geq 0$

$D \leq 0 \qquad 49-2a>0$

$(a-5)^2(a+12)(a-2) \leq 0$

$a \in [-12\ \ 2] \cup \{5\}$

$a < \frac{49}{2}$

$a \in (-12\ \ 2]$

Common root $\qquad (-12\ \ 2)$ Ans

(II) method

$(a-5)x^2 + (5-a) = 0$

$x^2 = \dfrac{\cancel{a-5}}{\cancel{a-5}}$

$x = \pm 1$

$f(1)\, f(-1) < 0$

$a \in (-12\ \ 2)$

9

Q $y = \dfrac{ax^2+3x+4}{4x^2+3x+a}$ $\quad y \in R. \quad a = ?$

Same $\underline{x = \pm 1}$

$(a+7)(a+1) < 0$

$\underline{(-7 \quad -1)}$

Q $y = \dfrac{x^2+x+2a}{x^2+2x+a}$ $\quad y \in R$ then find a.

$= -x + a = 0$

$\boxed{x = a}$

Common root par value Negative

$(a^2+a+2a) < 0$

$a(a+3) < 0$

$a \in (-3, 0)$

Identity

If a QE has three distinct real root then it becomes identity.

$ax^2+bx+c=0 \; \alpha, \beta, \gamma$

$a\alpha^2 + b\alpha + c = 0$

$a\beta^2 + b\beta + c = 0$

$a\gamma^2 + b\gamma + c = 0$

(I)-(II) $\quad a(\alpha^2-\beta^2) + b(\alpha-\beta) = 0$

$a(\alpha+\beta) + b = 0$

$\underline{a(\beta+\gamma) + b = 0}$ $\quad a = 0 \quad \alpha = \gamma$

but these are $\underline{\text{distinct}}$

$a(\alpha-\gamma) = 0$

Eqⁿ of the form $\underline{0x^2+0x+0=0}$

If any Polynomial eqⁿ becomes an identity then its all cofficient are simultaneously zero

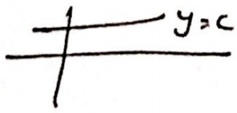
Q for what value of P the eqⁿ $(P+2)(P-1)x^2 + (P-1)(2P+1)x + P^2 - 1 = 0$ has more than 2 roots

Note QE with both ~~log~~ root zero
$\boxed{b=0}$ $\boxed{c=0}$

QE with one root zero
$\boxed{b \neq 0}$ $\boxed{c=0}$

both root infinity

x^2 and x cofficient should be 0



Condition for common Root :

* Both root Common

$$a_1x^2 + b_1x + c_1 = 0$$
$$a_2x^2 + b_2x + c_2 = 0$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

* one root common

$$\frac{\alpha^2}{b_1c_2 - b_2c_1} = \frac{\alpha}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\alpha = \frac{b_1c_2 - b_2c_1}{a_2c_1 - a_1c_2} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

$$(c_1a_2 - a_1c_2)^2 = (b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1)$$

$$\begin{array}{ccccccc} a_1 & & b_1 & & c_1 & & a_1 \\ & \times & & \times & & \times & \\ a_2 & & b_2 & & c_2 & & a_2 \end{array}$$

Q find the value of k for which the equation $3x^2+4kx+2=0$ and $2x^2+3x-2=0$ may have a common root

Q If the QE $x^2+bx+c=0$ and $x^2+cx+b=0$ have a Common root $(b \neq c)$ then Prove that their uncommon root of the eqn is $x^2+x+bc=0$

Q If $x^2+3x+5=0$ and $ax^2+bx+c=0$ have a Common root, $a,b,c \in N$ then value of $a+b+c$

Q If $4x^2\sin^2\theta - 4\sin\theta x+1=0$ and $a^2(b^2-c^2)x^2+b^2(c^2-a^2)x + c^2(a^2-b^2)=0$ have a common root and the second equation has equal root then find the Possible value of θ. where $\theta \in (0, \pi)$

Q If $x^2+ax+12=0$, $x^2+bx+15=0$ and $x^2+(a+b)x+36=0$ have a common +ve root, find a, b and common root.

Q If the equation $ax^2+bx+c=0$ and $x^3+3x^2+3x+2=0$ have a two common root. then Prove that $a=b=c$

$= (x+2)(x^2+x+1)$

$\underline{a=b=c=1}$

Resolving a Gen. Quadratic Expression in x and y into two linear factor:

$f(x,y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

$$x = \frac{-(hy+g) \pm \sqrt{(hy+g)^2 - a(by^2+2fy+c)}}{a}$$

$ax + hy + g = \pm\sqrt{y^2(h^2-ab) + 2y(hg-af) + g^2-ac}$

two linear factor only when quantity under radical sign is a Perfect square.

$D = 0$

$$\boxed{abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$$

Q factorise into linear factor $2x^2 + 3xy + y^2 + 2y + 3x + 1 \cdot$

$$x = \frac{-3(y+1) \pm \sqrt{(y+1)^2}}{4}$$

$2x + y + 1 = 0$ $\qquad$ $x + y + 1 = 0$

(1) Method $\qquad (2x + y + A)(x + y + B)$

Q factorise $x^2 - 3xy + 2y^2 - 2x - 3y - 35 = 0$

Q If the expression $x^2 + 16y^2 - 3x + 2 = 0$ is satisfied by the real value of x and y Prove that

$1 \le x \le 2$ and $-\frac{1}{8} \le y \le \frac{1}{8}$

Q x, y, z are three real no such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$ show that x, y, z lies on $\left[\frac{2}{3}, 2\right]$

$= z = 4 - x - y \qquad x^2 + y^2 + (4-x-y)^2 = 6$

Theory of Equation

✓ $ax^2+bx+c=0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix} = a(x-\alpha)(x-\beta)$

✓ $ax^3+bx^2+cx+d=0 \begin{smallmatrix}\alpha\\ \beta\\ \gamma\end{smallmatrix} : a(x-\alpha)(x-\beta)(x-\gamma)$

$\alpha+\beta+\gamma = -b/a \qquad \alpha\beta+\beta\gamma+\gamma\alpha = c/a \qquad \alpha\beta\gamma = -d/a$

✓ $ax^4+bx^3+cx^2+dx+e=0 = a(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

A Polynomial eqⁿ of degree odd with real cofficient must have atleast one real root as imaginary root always occour in pair of conjugate

Q. Solve the cubic $4x^3+16x^2-9x-36=0$ the sum of its root being equal to 0.

Q. $\alpha, \beta, \gamma, \delta$ are the roots of the eqⁿ $\tan(\frac{\pi}{4}+x) = 3\tan 3x$ no two of which have equal tangent then find $\tan\alpha + \tan\beta + \tan\gamma + \tan\delta$

Note $a_n x^n + a_{n-1}x^{n-1}$ ——— $a_1 x + a_0 = \begin{smallmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{smallmatrix}$

$$\sum_{i=1}^{n} x_i = \frac{-a_{n-1}}{a_n}$$

$$\sum x_1x_2 = \frac{+a_{n-2}}{a_n}$$

$$\sum x_1x_2x_3 = \frac{-a_{n-3}}{a_n}$$

$$\vdots$$

$$\prod x_1 = \frac{(-1)^n a_0}{a_n}$$

Q If α, β, γ are the roots of the eqn $x^3 + 2x^2 - 3x + 1 = 0$ than find $\sum \frac{\alpha\beta}{\alpha+\beta}$

$= \frac{\alpha\beta}{\alpha+\beta} = \frac{1}{\frac{1}{\beta}+\frac{1}{\alpha}}$ $\qquad \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = 3$

$\frac{1}{3-\frac{1}{\gamma}} = \frac{\gamma}{3\gamma-1}$

$\frac{\gamma}{3\gamma-1} = t$ $\qquad \gamma = \frac{t}{3t-1}$

$\left(\frac{t}{3t-1}\right)^3 + 2\cdot\left(\frac{t}{3t-1}\right)^2 - 3\left(\frac{t}{3t-1}\right) + 1 = 0$

Q Let α, β, γ are the root of $x^3 + 2x^2 + 3x + 3 = 0$ find

(i) $\sum \frac{\alpha}{\alpha+1}$

(ii) $\sum \left(\frac{\alpha}{\alpha+1}\right)^3$

(i) $\frac{\alpha}{\alpha+1} = t$

$\alpha = \alpha t + t$ $\qquad \alpha = \frac{t}{1-t}$

$\left(\frac{t}{1-t}\right)^3 + 2\left(\frac{t}{1-t}\right) + 3\left(\frac{t}{1-t}\right) + 3 = 0$

$-t^3 + 5t^2 - 6t + 3 = 0$ t_1, t_2, t_3

$\sum t_1 = \frac{-5}{-1} = 5$

(ii) $\sum t_1^3 + t_2^3 + t_3^3$

$= 3t_1t_2t_3 + (t_1+t_2+t_3)\left(t_1^2+t_2^2+t_3^2 - 3(t_1+t_2+t_3)\right]$

$= \boxed{44}$

Q Solve $6x^3 - 11x^2 + 6x - 1 = 0$ If the roots of Eqⁿ are in H.P. (12)

Put $x = \frac{1}{y}$ $\quad y^3 - 6y^2 + 11y - 6 = 0$ root are in A.P.

$\alpha - \beta, \; \alpha, \; \alpha + \beta$

$\underline{\alpha = 2}$ $\qquad \beta = \pm 1$

$(1 \;\; 2 \;\; 3) \qquad (3 \;\; 2 \;\; 1)$

roots of given Eqⁿ $\left(1, \frac{1}{2}, \frac{1}{3}\right) \quad \left(\frac{1}{3}, \frac{1}{2}, 1\right)$

Q $\alpha, \beta, \gamma, \delta$ are the roots of Eqⁿ $x^4 + 4x^3 - 6x^2 + 7x - 9 = 0$ then roots of

$(1+\alpha^2)(1+\beta^2)(1+\gamma^2)(1+\delta^2)$

$= x^4 + 4x^3 - 6x^2 + 7x - 9 = (x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

Put $x = i, \quad x = -i$

Q $x^4 - 3x^3 + x^2 - 2x + 1 = 0$ $\alpha, \beta, \gamma, \delta$

find $\alpha^3 + \beta^3 + \gamma^3 + \delta^3$

$= \alpha^4 - 3\alpha^3 + \alpha^2 - 2\alpha + 1 = 0$ $\qquad \alpha$ se divide

$= \alpha^3 - 3\alpha^2 + \alpha - 2 + \frac{1}{\alpha} = 0$

$= \alpha^3 = 3\alpha^2 - \alpha + 2 - \frac{1}{\alpha}$

$= \Sigma\alpha^3 = 3\Sigma\alpha^2 - \Sigma\alpha + \Sigma 2 - \Sigma\frac{1}{\alpha}$

$= \; = 3\left[(\Sigma\alpha)^2 - 2\Sigma\alpha\beta\right] - (\alpha+\beta+\gamma+\delta) + (2+2+2+2) - \left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}\right)$

$= \; 24$

Q α, β, γ are the roots of $x^3 - 2x + 3 = 0$ and $f(x) = a^x + b^x + c^x$

then find the value of $\frac{f(10) + 3f(7)}{f(8)}$

$a^3 - 2a + 3 = 0$ $\quad a^7$ multiply

$a^{10} - 2a^8 + 3a^7 = 0$

$b^{10} - 2b^8 + 3b^7 = 0$ $\quad$ add

$c^{10} - 2c^8 + 3c^7 = 0$

$a^{10} + b^{10} + c^{10} - 2(a^8 + b^8 + c^8) + 3(a^7 + b^7 + c^7) = 0$

$f(10) - 2f(8) + 3.f(7) = 0$

$\frac{f(10) + 3f(7)}{f(8)} = 2$ Ans

Q form a cubic equation whose roots are the cube of the root of Eq^n $x^3 + 3x^2 + 2 = 0$ α, β, γ

= wo Eq^n find karni $\alpha^3, \beta^3, \gamma^3$

$\alpha^3 = t$

$\alpha = t^{1/3}$

$\alpha^3 + 3\alpha^2 + 2 = 0$

$(t^{1/3})^3 + 3.(t^{1/3})^2 + 2 = 0$

$t + 3.t^{2/3} + 2 = 0$

$t + 2 = -3.t^{2/3}$ $\quad$ cube side

$t^3 + 33t^2 + 12t + 8 = 0$

Q If α, β are the root of $ax^2 + bx + c = 0$ then find the eq^n $a(x-3)^2 + b(x-3)(x+2) + c(x-2)^2 = 0$ in term of α, β.

$= a\left(\frac{x-3}{x-2}\right)^2 + b\left(\frac{x-3}{x-2}\right) + c = 0$ $\quad$ (divide $(x-2)^2$)

$= \frac{x-3}{x-2} = \alpha$ $\quad \frac{x-3}{x-2} = \beta$

$= x = \frac{3-2\alpha}{1-\alpha}, \frac{3-2\beta}{1-\beta}$

Q If α, β, γ are the root of $x^3 + 2x - 3 = 0$ then find the equation whose root are $(\alpha-\beta)(\alpha-\gamma)$, $(\beta-\gamma)(\beta-\alpha)$, $(\gamma-\alpha)(\gamma-\beta)$.

$= \alpha^2 - \alpha(\beta+\gamma) + \beta\gamma$

$\alpha+\beta+\gamma = 0$ $\alpha\beta\gamma = 3$ $\Sigma\alpha\beta = 2$

$\alpha^2 - \alpha(-\alpha) + \frac{3}{\alpha})$

$\alpha^2 + \alpha^2 + \frac{3}{\alpha}$ $\frac{2\alpha^3+3}{\alpha}$ $\alpha^3 = 3 - 2\alpha$

$\frac{6-4\alpha+3}{\alpha}$ $\frac{9-4\alpha}{\alpha} = t$

$\alpha = \frac{9}{t+4}$

$= \left(\frac{9}{t+4}\right)^3 + 2\left(\frac{9}{t+4}\right) - 3 = 0$

* **Location of Root**

$f(x) = ax^2 + bx + c$

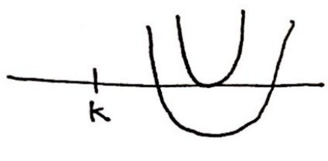
① When both root of $f(x) = 0$ are greater than a specfied no K.

$a > 0$ $a > 0$

$D \geq 0$

$f(K) > 0$

$-b/2a > K$

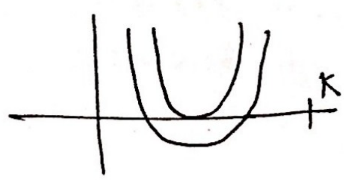


Q find all the value Parameter d for which both root of E_q^n $x^2 - 6dx + 2 - 2d + 9d^2 = 0$ exceed than 3

② Both the root are less than K

$a > 0$ $D \geq 0$

$f(K) > 0$ $-b/2a < K$

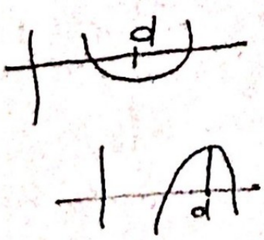


Q. Find the range of α for which the Eq^n $x^2 + 4\alpha x + 4\alpha^2 - 3\alpha + 2 = 0$ for the Eq^n has
1. Both root +ve
2. Both root less than 1.

Type-3 Both root of QE lies on either side of given no.

$a > 0$ $\quad$ $a < 0$

$f(\alpha) < 0$ $\quad$ $f(\alpha) > 0$



$$\boxed{a f(\alpha) < 0}$$

Q. Find the value of K for which one root of the Eq^n $x^2 - (K+1)x + K^2 + K - 8 = 0$ exceed 2 and other is smaller than 2.

Q. Find the set of all values of a for which roots of Polynomial $(a^2 + a + 1)x^2 + (a-1)x + a^2$ are located on either side of 3.

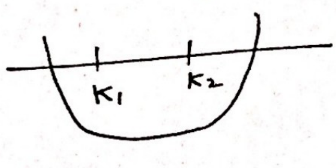
Type-4 $\quad f(x) = ax^2 + bx + c$

$f(x) = 0$

K_1 and $K_2 \in (\alpha, \beta)$ $\quad$ where α, β are the root of $f(x) = 0$

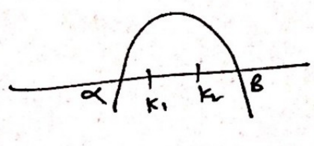
$a > 0$

one root is greater and one is less.



$a f(K_1) < 0$ $\quad \cap$

$a f(K_2) < 0$



Q. Find the values of K for which one root of QE $(K-5)x^2 - 2Kx + K - 4 = 0$ is smaller than 1 and other exceed 2.

$K-5>0 \quad K>5$

$f(1)<0$

$f(2)<0$

$K \in (5\ 24)$

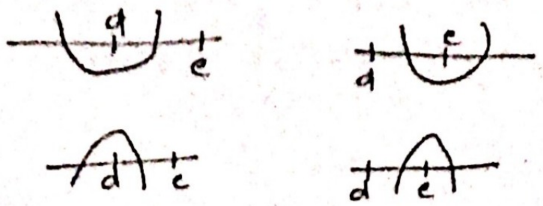
$K-5<0 \quad K<5$

$f(1)>0 \quad f(2)<0$

$= (5\ 24)$ Ans

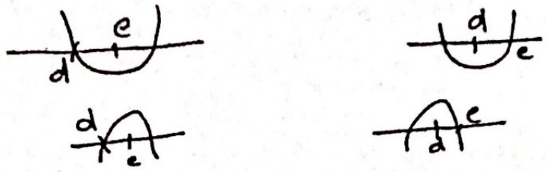
(T-5) $f(x) = ax^2+bx+c$ Exactly one root lie in $(d\ e)$

$d<e$



$a \neq 0 \quad f(d)f(e)<0$ and also check end Points

$f(d)=0$ and $f(e)=0$



Q Find the set of values of m for which exactly one root of the eqn $x^2+mx+m^2+6m=0$ lie

(I) $(-2,0)$ (II) $[-2\ 0)$ (III) $[-2\ 0]$ (IV) $(-2\ 0]$

$= \quad f(-2)f(0)<0$

$m \in (-6\ -2) \cup (-2\ 0)$

$f(-2)=0$

$(m+2)^2=0 \qquad x^2-2x-8=0$

$m=-2 \qquad 4; -2$

$f(0)=0 \quad m^2+6m=0 \qquad x^2=0 \quad (0\ 0)$

$m(m+6)\Rightarrow \ 0, -6 \qquad x^2-6x=0 \quad (0,6)$

$m = -2$

(I) $m \in (-6\ -2) \cup (-2\ 0)$

(II) $m \in (-6\ 0)$

(III) $m \in [-6\ 0)$

(IV) $m \in [-6\ -2) \cup (-2\ 0)$

Q find a for which exactly one root of the eqn
$x^2 - (a+1)x + 2a = 0$

(I) $(0\ 3)$ (II) $[0\ 2)$ (III) $(0\ 3]$ (IV) $[0\ 3]$

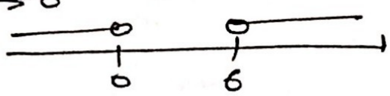
$= f(0)\, f(3) < 0$ also check
$f(0) = 0$
$f(3) = 0$

$= 2a(9 - (a+1)3 + 2a) < 0$

$= \cancel{2a(9 - 3a - 3 + 2a) < 0}$

$= \cancel{-6a < 0}$ $(2a)(6-a) < 0$

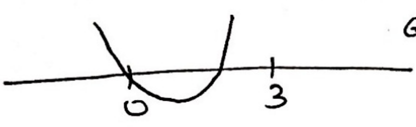
$= \cancel{a > 0}$



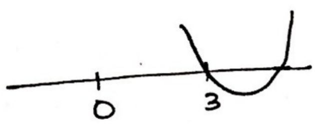
$f(0) = 0$ $a = 0$ $x^2 - x = 0$ $x = 0, 1$

$f(3) = 0$ $a = 6$ $x^2 - 7x + 12 = 0$ $x = 3, 4$

$a = 0$



$a = 6$



(I) $a \in (-\infty\ 0] \cup (6\ \infty)$

(II) $a \in (-\infty\ 0) \cup (6\ \infty)$

(III) $a \in (-\infty\ 0] \cup [6\ \infty)$

(IV) $a \in (-\infty\ 0) \cup [6\ \infty)$

T-6 $f(x) = ax^2 + bx + c = 0$

$f(d) \cdot f(e) < 0$

than exactly one root lies in the interval $(d\ e)$

Q $f(x) = ax^2 + bx + c$

and $(a-b+c)(a+2b+4c) < 0$

than commend on the root of $f(x)$

= $f(-1) \cdot 4\left(\frac{a}{4} + \frac{b}{2} + c\right) < 0$

$4 f(-1) \cdot f(\frac{1}{2}) < 0$

real distinct and one root will lie

between -1 and $\frac{1}{2}$.

T-7 $f(x) = ax^2 + bx + c$

let $f(x) = 0$ has root α and β then

$\alpha, \beta \in (d\ e)$

$a > 0$

$f(d) > 0$

$f(e) > 0$

$D \geq 0$ $\quad d < -\frac{b}{2a} < e$

Q If α and β are the root of Eq^n $x^2 + 2(k-3)x + 9 = 0$ $(\alpha \neq \beta)$ $\alpha, \beta \in (-6\ 1)$ than find k.

Q If α, are real root of $ax^2 + bx + c = 0$ — α and β be the real root of the Eq^n $-ax^2 + bx + c = 0$ — β $(\alpha \neq \beta)$ then show that their exist a real no y satisfied the eq^n $\frac{a}{2}x^2 + bx + c = 0$ lie between α and β. $y \in (\alpha, \beta)$

= Let $f(x) = \frac{a}{2}x^2 + bx + c$

$f(\alpha) \cdot f(\beta) < 0$ (Agar Aisa Prove kar du to

$a\alpha^2 + b\alpha + c = 0$ (I)

$-a\beta^2 + b\beta + c = 0$ (II)

$f(\alpha)\cdot f(\beta)$

$\left(\frac{a}{2}\alpha^2 + b\alpha + c\right)\left(\frac{a}{2}\beta^2 + b\beta + c\right)$

$\left(\frac{a}{2}\alpha^2 - a\alpha^2\right)\left(\frac{a}{2}\beta^2 + a\beta^2\right)$

$\left(-\frac{a}{2}\alpha^2\right)\left(\frac{3a}{2}\beta^2\right)$

$\underline{-\frac{3}{4}a^2\alpha^2\beta^2 < 0}$

Q. find the all real value of m

$f(x) = mx^2 - 4x + 3m + 1 > 0$

(I) $\forall\, x \in R$

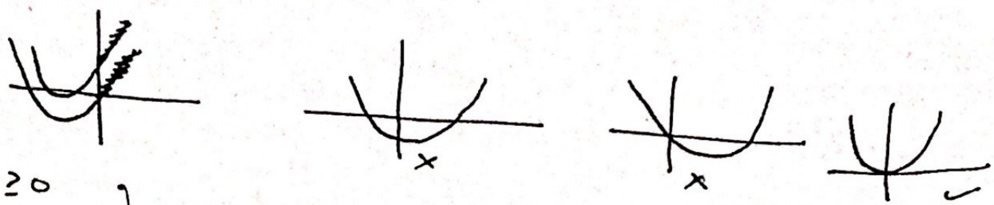
(II) $\forall$ +ve x (all +ve x)

(I) $m > 0$ $\quad d < 0$

(II) Case-1 $m = 0$ $\quad -4x + 1 > 0$ Not Possible

Case-2 $m < 0$ Not Possible

Case-3 $m > 0$



$\left.\begin{array}{l} D \geq 0 \\ -b/2a \leq 0 \\ f(0) \geq 0 \end{array}\right\} \cap$ or $D < 0$

$m \in \left[-\frac{1}{3}, 0\right) \cup (1, \infty)$
X

$\underline{m > 0}$ $\quad (1, \infty)$ ✓

Q. Let $x^2 - (m-3)x + m = 0$ $(m \in R)$ be a Q.E. find the value of m for which

(I) Both root are smaller than 2

(II) ——— greater 2

(III) one root is smaller than 2 and other root is greater than 2

(4) Exactly one root lies in the interval (1 2)

(5) Both the root lies in the interval (1 2)

(6) one root is greater than 2 and other root is smaller than 1.

Q If $a<b<c<d$ then show that the QE.
$(x-a)(x-c) + 2(x-b)(x-d) = 0$ has real root

Q $f(x) = m.2^{2x} - 4.2^x + 3m + 1 > 0 \quad \forall x \in R$ find m

$= 2^x = t > 0$

$g(t) = m.t^2 - 4t + 3m + 1 > 0 \quad \forall t > 0$

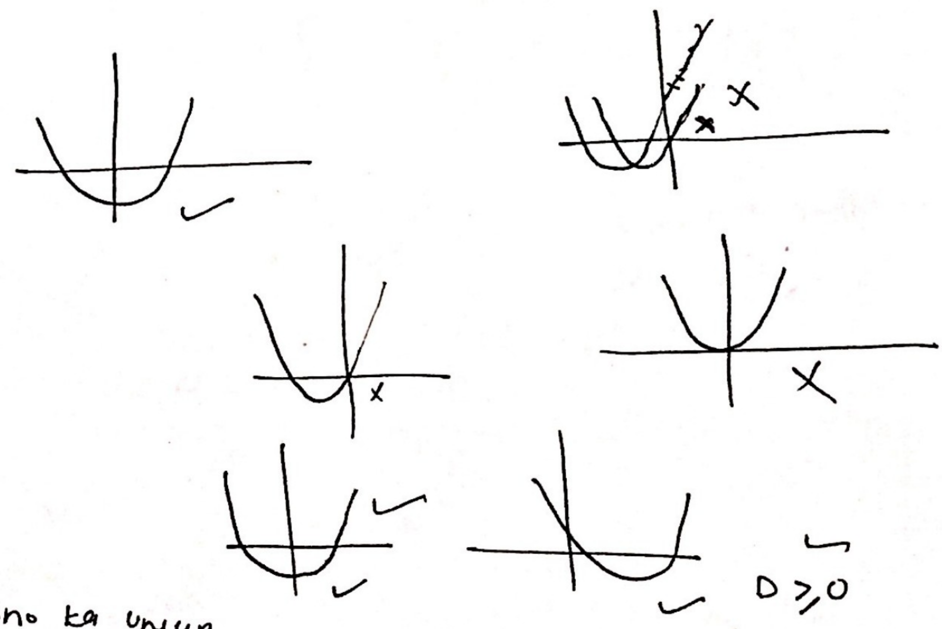
same question

Q $f(x) = m\tan^2 x - 4\tan x + 3m + 1 > 0 \quad \forall x \in (0 \frac{\pi}{2})$

let $\tan^2 x = t$

$g(t) = mt^2 - 4t + 3m + 1 > 0 \quad \forall t > 0$

Q find the value of a for which $x^2 - ax - a + 3 \leq 0$ for atleast one +ve x



$f(0) < 0$

dono ka union

$D \geq 0$

$-b/2a > 0$

$f(0) \geq 0$

Q: Find the value of K for which the equation
$x^4 + (1-2K)x^2 + K^2 - 1 = 0$ is satisfied

① No real value of x.

② Exactly one real value of x

③ ——— distinct two ———

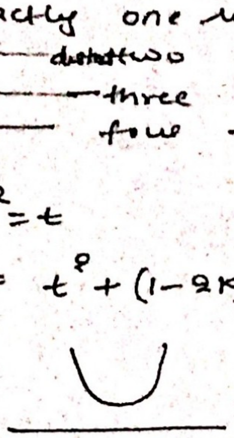
④ ——— three ———

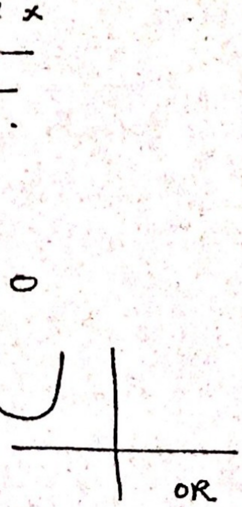
⑤ ——— four ———.

Let $x^2 = t$

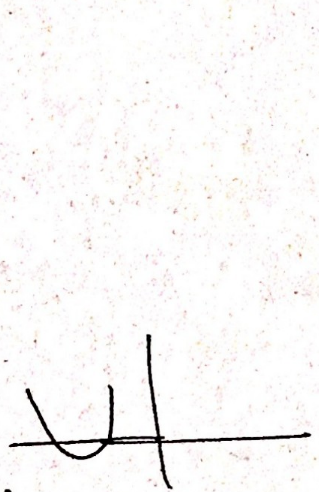
$f(t) = t^2 + (1-2K)t + K^2 - 1 = 0$

①





OR



$x^2 = t <^{-ve}_{-ve}$ Nahi

$D < 0$ OR

$(1-2K)^2 - 4(K^2-1) < 0$

$K > \frac{5}{4}$

$\left.\begin{array}{l} D \geq 0 \\ f(0) > 0 \\ -\frac{b}{2a} < 0 \end{array}\right]$ intersection

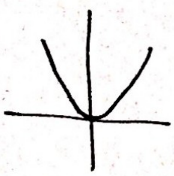
dono Result ka unions $K < -1$

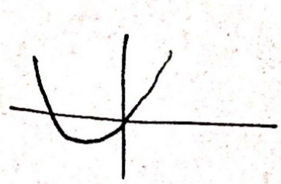
$= K < -1 \cup K > \frac{5}{4}$

$= \boxed{K \in (-\infty, -1) \cup (\frac{5}{4}, \infty)}$

② either $x^2 = t <^{0}_{0}$

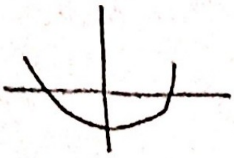
$x^2 = t <^{0}_{-ve}$



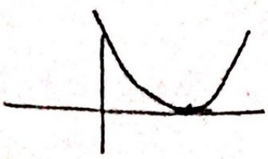


$\left.\begin{array}{l} D \geq 0 \\ -\frac{b}{2a} \leq 0 \\ f(0) = 0 \end{array}\right] \cap$

(3) $x^2 = t$ < +ve, −ve $x^2 = t$ < +, + Equal



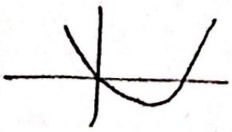
$f(0) < 0$



$\cup$

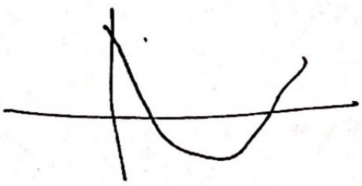
$$\left.\begin{array}{l} D = 0 \\ f(0) > 0 \\ -b/2a > 0 \end{array}\right] \cap$$

(4) $x^2 = t$ < +ve, zero



$$\left.\begin{array}{l} D > 0 \\ -b/2a > 0 \\ f(0) = 0 \end{array}\right] \cap$$

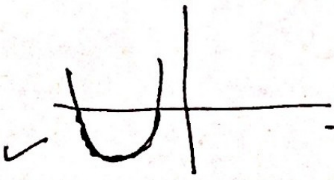
(5) $x^2 = t$ < +ve, +ve distinct

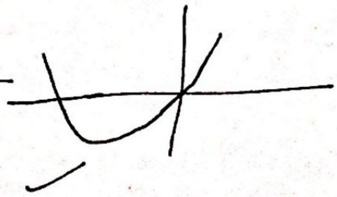


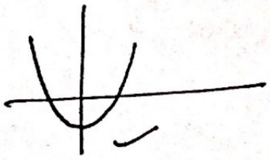
$$\left.\begin{array}{l} D > 0 \\ f(0) > 0 \\ -b/2a > 0 \end{array}\right] \cap$$

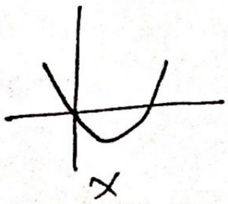
Q find the value of a for which $(a^2+3)x^2 + (a+2)x - 5 < 0$ for atleast one −ve x.

=

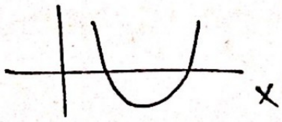








$$\left.\begin{array}{l} D > 0 \\ f(0) \geq 0 \\ -b/2a < 0 \end{array}\right] \cap \quad \cup \quad f(0) < 0$$



Q. find the value of a for which $(a^2+2)x^2+(\sqrt{5a+3})x-\frac{1}{4}<0$ is satisfied for atleast one real x.

= 0 ke uye to dikh hi raha hai.

$a \in R$ But Problem ho gayi domain ka dyan rakhna hau..

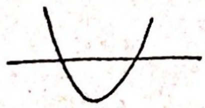
$5a+3 \geq 0$

$\boxed{a \geq -3/5}$

common

$\boxed{a \geq -\frac{3}{5}}$

Q. Find the value of p for which $x^2-2px+3p+4<0$ for atleast one x.



$D>0$

$4p^2-4(3p+4)>0$

$p^2-3p-4>0$

$(p-4)(p+1)>0$

Q. Find the value of m for which every solution of inequation $1 \leq x \leq 2$ is the solution of inequation $x^2-mx+1<0$

$f(1)<0$ $\cap$
$f(2)<0$

Q If e^{α} and $e^{-\alpha}$ are the roots of the equation $3x^2 - (a+b)x + 2a = 0$ $a, b \in R$ $\alpha \neq 0$ then least integral value of b is. (18)

$e^{\alpha} \cdot e^{-\alpha} = \frac{2a}{3}$

$\boxed{a = 3/2}$

$e^{\alpha} + e^{-\alpha} = \frac{a+b}{3}$

> 2

$\frac{a+b}{3} > 2$

$a+b > 6$

$3/2 + b > 6$

$b > 6 - 3/2$

$b > 9/2$

(5)

Q Let x_1 and x_2 be the roots of the QE $x^2 + px + q = 0$ If $x_1 = \frac{x_2 + 4}{2x_2 - 1}$, then value of $(2q + p)$ is

$= 2x_1x_2 - x_1 = x_2 + 4$

$= 2q = -p + 4$

$= \boxed{2q + p = 4}$

Q If α, β are the roots of the QE. $x^2 + 2(1 - \cos 3\theta)x - 2\sin^2 3\theta = 0$ $(\theta \in R)$, then maximum value of $\alpha^2 + \beta^2$

$= \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 4(1 - \cos 3\theta)^2 + 2 \cdot 2\sin^2 3\theta$

$= 8 - 8\cos 3\theta$

$= 8 - 8[-1]$

$= \underline{16}$

Q The smallest integral value of P for which the inequality $(P-3)x^2 - 2Px + 3(P-2) > 0$ is satisfied for all real value of x.

$P - 3 > 0$ $D < 0$

Q Let $f(x) = x^2 + px - 2$, $g(x) = px^2 + x + (p+2)$ $\forall x \in R$ where p is a real constant. If $f(x) > g(x)$ $\forall x \in R$ then range of p is $(-\infty, -\frac{m}{n})$, where m and n are coprime. Find the value of $(m - 5n)$

$= x^2 + px - 2 > px^2 + x + (p+2)$

$= (1-p)x^2 + (p-1)x - (4+p) > 0$ $\forall x \in R$

$\boxed{p = 1 \quad \times}$ Not satisfied $\quad p \neq 1$

$1 - p > 0 \qquad D < 0$

$(p-1)^2 + 4\cdot(1-p)(4+p) < 0$

$(p-1)(-3p-17) < 0$

$p \in (-\infty, -\frac{17}{3})$

$17 - 15 = 2$

Q If the root of the equation $x^2 + (p+1)x + 2q - q^2 + 3 = 0$ are real and unequal $\forall p \in R$, then find the minimum integral value of $(q^2 - 2p)$

$= D > 0$

$= (p+1)^2 - 12 > 4(2q - q^2)$

$= 4(q^2 - 2q) > 12 - (p+1)^2$

$= \boxed{q^2 - 2q > 3}$

Q If two roots of the equation $(x-1)(2x^2 - 3x + 4) = 0$ coincide with roots of the equation $x^3 + (a+1)x^2 + (a+b)x + b = 0$ where $a, b \in R$ then $2(a+b)$ is

$= x^3 + ax^2 + x^2 + ax + bx + b = 0$

$= x^2(x+1) + ax(x+1) + b(x+1) = 0$

$= (x+1)(x^2 + ax + b) = 0$

x^2+ax+b and $2x^2-3x+4$ both root common

$\boxed{a=-\frac{3}{2} \quad b=2}$

Q. If $\sin^2 x+\sin x = a+2$, then which of following statement is are correct.

✓ a) No of integral values of a for real solution to exist is 3.

✓ b) There exist no solution for $a<-\frac{9}{4}$ or $a>0$

✗ c) minimum value of a for real solution is -2

✗ d) no of prime values of a for real solution to exist is 1

$-1 \le t \le 1$

$t^2+t = a+2$

$\left(t+\frac{1}{2}\right)^2-\frac{1}{4} = a+2$

$a+2 \ge -\frac{1}{4}$

$-\frac{1}{4} \le a+2 \le 2$

$\boxed{-\frac{9}{4} \le a \le 0}$ $\quad \{-2,-1,0\}$

Q. If the equation $x^3+2x^2-4x+5=0$ has root α, β, γ, then value of $\dfrac{(\alpha^3+5)(\beta^3+5)(\gamma^3+5)}{13\alpha\beta\gamma}$

$=\frac{1}{13}\left[\prod\left(\frac{\alpha^3+5}{\alpha}\right)\right]$

$\alpha^3+2\alpha^2-4\alpha+5$

$\alpha^3+5 = 4\alpha-2\alpha^2$

$\frac{\alpha^3+5}{\alpha} = 4-2\alpha$

$=\frac{1}{13}\left[\prod(4-2\alpha)\right]$

$=\frac{2\times2\times2}{13}\left[(2-\alpha)(2-\beta)(2-\gamma)\right]$

$x^3+2x^2-4x+5 = (x-\alpha)(x-\beta)(x-\gamma)$

$8+\cancel{8}-\cancel{8}+5 = (2-\alpha)(2-\beta)(2-\gamma)$

$=\frac{8\times\cancel{13}}{\cancel{13}}$

$= 8$

$= 5\sum \alpha^3 - 3\alpha$

Q. If the general expression of degree 2 given by $3x^2 + xy + ky^2 + 10x - 3y + 7$ can be factorised into two linear factor then value of k is

Q. If α, β, γ are the roots of cubic Equation $x^3 - 3x^2 + 2x + 4 = 0$ and

$$y = 1 + \frac{\alpha}{x-\alpha} + \frac{\beta x}{(x-\alpha)(x-\beta)} + \frac{\gamma x^2}{(x-\alpha)(x-\beta)(x-\gamma)}$$

then value of y at x = 2 is

$$= y = \frac{x}{x-\alpha} + \frac{\beta x}{(x-\alpha)(x-\beta)} + \frac{\gamma x^2}{(x-\alpha)(x-\beta)(x-\gamma)}$$

$$y = \frac{x^3}{(x-\alpha)(x-\beta)(x-\gamma)}$$

$$y = \text{at } x = 2 \quad \frac{8}{(2-\alpha)(2-\beta)(2-\gamma)}$$

$$x^3 - 3x^2 + 2x + 4 = (x-\alpha)(x-\beta)(x-\gamma)$$

$$\cancel{8} - \cancel{12} + \cancel{4} + 4 = (2-\alpha)(2-\beta)(2-\gamma)$$

$$4$$

$$y = \frac{\cancel{8}}{\cancel{4}} \quad 2$$

Q. If α, β, γ are the roots of the Equation $5x^3 - qx - 1 = 0$ $(q \in R)$ then find the value of

$$\frac{\alpha^2 - 3}{\beta\gamma} + \frac{\beta^2 - 3}{\gamma\alpha} + \frac{\gamma^2 - 3}{\alpha\beta}$$

$$= 5\alpha^3 - q\alpha - 1 = 0$$

$$= \frac{\alpha(\alpha^2 - 3)}{\alpha\beta\gamma} + \frac{\beta(\beta^2 - 3)}{\alpha\beta\gamma} + \frac{\gamma(\gamma^2 - 3)}{\alpha\beta\gamma}$$

$= 5\sum \alpha^3 - 3\alpha$

$= \sum 5\alpha^3 - 15\alpha$

$= \sum 5\alpha^3 - 15\sum\alpha$ $\qquad \underline{\alpha+\beta+\gamma=0}$

$= \sum 5\alpha^3$

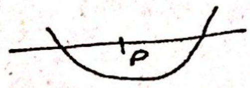
$= \sum 9\alpha+1$

$= 9(\alpha+\beta+\gamma) + \sum 1$

$=$ 3

Q. The range of $P \in R$ for which the equation $2x^2 - 2(2P+1)x + P(P+1) = 0$ have one root less than p and other root greater than p is

$f(p) < 0$



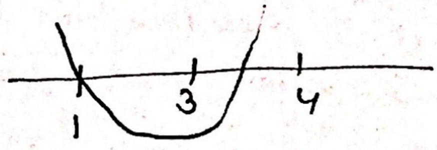
$2p^2 - 2(2p+1)p + p(p+1) < 0$

$p < -1$ or $p > 0$

Q. If the equation $x^2 + Ax + B = 0$ has one root equal to unity and other root lies between roots of the equation $x^2 - 7x + 12 = 0$ then range of a.

$= \underline{3, 4}$

$f(x) = x^2 + Ax + B$



$$\left.\begin{array}{l} f(1) > 0 \\ f(3) < 0 \\ f(4) > 0 \end{array}\right\} \cap$$

$a < -4$

Q. If $\alpha, P \in R$ and $P \in [5\ 10]$ then the no of integral value of P for which $e^{\alpha}+1$ and $e^{-\alpha}+1$ are the root of QE $x^2 + (1-2P)x + 2P - 1 = 0$ is

$e^{\alpha}+1,\ e^{-\alpha}+1$, means both root are greater than 1.

$D \geq 0$

$-\frac{b}{2a} > 1$

$f(1) > 0$

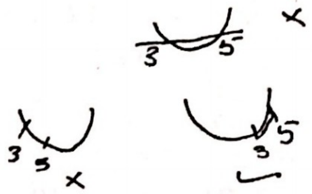
- total 8 values.

Q. Let a, b be arbitrary real no. Find the smallest natural no b for which the equation $x^2 + 2(a+b)x + (a-b+8) = 0$ has unequal real root for $a \in R$.

Q. Let $P(x) = (m^2+4m+5)x^2 - 4x + 7$, $m \in R$ if $3 \leq x \leq 5$, then find minimum the minimum value of $P(x)$

$-\frac{b}{2a}$

$\frac{4}{2(m^2+4m+5)}$



$x = \frac{2}{(m+2)^2+1} \leq 2$ minimum at $x=3$

$P(x) = (m^2+4m+5)9 - 12 + 7$

$P(x) = 9(m+2)^2 + 4$

$\boxed{Ans = 4}$

Q. Let α, β, γ are the root of the cubic $x^3 - 3x^2 + 1 = 0$. Find a cubic whose roots are

$\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}, \frac{\gamma}{\gamma-2}$.

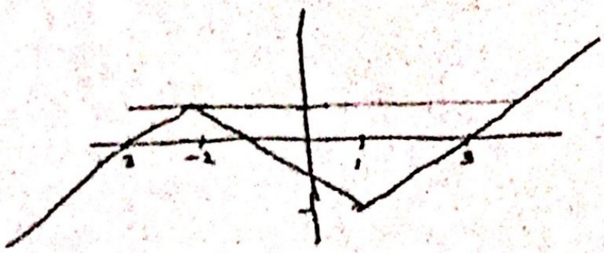
Q. If the equation $|1-x| - |x+2| + x = p$ has two distinct real solution then $p \in$ to

$y = |1-x| - |x+2| + x \qquad y = p$

$$y = \begin{cases} x-3 & x \geq 1 \\ -x-1 & -2 \leq x < 1 \\ x+3 & x < -2 \end{cases}$$

$y = 1$

$y = -2$



$p \in \{1, -2\}$

Q. The smallest integral value of α for which the inequality $1 + \log_5(x^2+1) \le \log_5(\alpha x^2 + 4x + \alpha)$ is true for all $x \in R$ is

$= \log_5(5x^2 + 5) \le \log_5(\alpha x^2 + 4x + \alpha)$

$= 5x^2 + 5 \le \alpha x^2 + 4x + \alpha$

$= (5-\alpha)x^2 - 4x + 5 - \alpha \le 0$

$5 - \alpha < 0$ $\qquad D \le 0$

$\underline{\alpha > 5}$

$16 - 4\cdot(5-\alpha)^2 \le 0$

$(\alpha - 3)(7 - \alpha) \le 0$

$(-\infty\ 3] \cup [7\ \infty)$

$\alpha \in [7\ \infty)$

Q. The set of values of a for which the equation $\cos 2x + a \sin x = 2a - 7$ posses a solution if

$= 1 - 2\sin^2 x + a \sin x - 2a + 7 = 0$

$= 2(4 - \sin^2 x) - a(2 - \sin x) = 0$

$= 2(2 + \sin x)(2 - \sin x) - a(2 - \sin x) = 0$

$= (2 - \sin x)(4 + 2\sin x - a) = 0$

$= 2\sin x + 4 - a = 0$

$= a = 4 + 2\sin x$

$a = 4 + 2[-1\ 1]$

$= 4 + [-2\ 2]$

$= [2\ 6]$

Q. If the root of the cubic $x^3 + ax^2 + bx + c = 0$ are three consecutive Positive integer, then the value of $\frac{a^2}{b+1}$ is equal to

$= n-1, n, n+1$

$= 3n = -a \quad = \quad 9n^2 = a^2$

$= (n-1)n(n+1) = -c$

$(n-1)n + n(n+1) + (n+1)(n-1) = b$

$n^2 - \cancel{n} + n^2 + \cancel{n} + n^2 - 1 = b$

$3n^2 = b+1$

$9n^2 = 3b+3$

$a^2 = 3b+3$

$a^2 = 3(b+1)$

$$\boxed{\frac{a^2}{b+1} = 3}$$